

Variable cosmological term $\Lambda(t)$.

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We present the case of time-varying cosmological term $\Lambda(t)$. The main idea arises by proposing that as in the cosmological constant case, the scalar potential is identified as $V(\phi) = 2\Lambda$, with Λ a constant, this identification should be kept even when the cosmological term has a temporal dependence, i.e., $V(\phi(t)) = 2\Lambda(t)$. We Use the Lagrangian formalism for a scalar field ϕ with standard kinetic energy and arbitrary potential $V(\phi)$ and apply this model to the Friedmann-Robertson-Walker (FRW) cosmology. Exact solutions of the field equations are obtained by a special ansatz to solve the Einstein-Klein-Gordon equation and a particular potential for the scalar field and barotropic perfect fluid. We present the evolution on this cosmological term with different scenarios.

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I. INTRODUCTION

The present phase of an accelerated expansion of the universe stands as one of the most challenging open problems in modern cosmology and astrophysics. This acceleration is characterized by which is popularly known as dark energy. Among many possible alternatives, the simplest candidate for dark energy is the vacuum energy which is mathematically equivalent to the cosmological constant. Models with different decay laws for the variation of cosmological term were investigated during the last two decades in a non covariant way, (Chen & Wu 1990); (Abdel 1990); (Pavon 1991); (Carvalho et al 1992); (Kalligas et al 1992); (Lima & Maia 1994); (Lima & Carvalho 1994); (Lima & Trodden 1996); (Arbab & Abdel 1994); (Birkel & Sarkar 1997); (Silveira & Waga 1997); (Starobinsky 1998); (Overduin & Cooperstock 1998); (Vishwakarma 2000,2001); (Arbab 2001,2003,2004); (Cunha & Santos 2004); (Carneiro & Lima 2005); (Fomin et al 2005); (Sola & Stefancic 2005,2006); (Pradhan et al 2007); (Jamil & Debnath 2011) and (Mukhopadhyay 2011); in particular, in Fomin et al (2005) there are several evolution relations for Λ which many author have used, also in Ref. Overduin & Cooperstock (1998) appears a table with these relations and the corresponding references where they were considered. Anisotropic cosmological models, also has been treated in this formalism from different points of view (Aroonkumar 1993,1994); (Arbab 1997); (Singh et al 1998); (Pradhan & Kumar 2001); (Pradhan 2003,2007,2009); (Pradhan & Pandey 2003,2006); (Pradhan et al 2007,2008); (Carneiro 2005); (Esposito et al 2007); (Bal & Singh 2008); (Belinchón 2008); (Singh et al 2008), (Singh et al 2013); (Shen 2013); (Tripathy 2013) and (Rahman & Ansary 2013).

In this work we present an analysis in covariant way, using the Lagrangian density of standard scalar field. The main idea arises by proposing that as in the cosmological constant case, the scalar potential is identified as $V(\phi) = 2\Lambda$, with Λ a constant. So, we *extend* this idea and suggest that this correspondence is valid even when this cosmological term has a temporal dependence, i.e., $V(\phi(t)) = 2\Lambda(t)$. We include a barotropic equation state between the pressure

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and energy density of the scalar field, $p_\phi = \omega_\phi \rho_\phi$, quantities that we shall define in the following lines. In order to built up the analysis presented here, initially we solve the Klein-Gordon equations, whose solution implies that the energy density of a scalar field has a wide range of scaling behavior, $\rho_\phi \sim A^{-m}$ with A the scale factor of the FRW model, (Ferreira & Joyce 1998); (Liddle & Sharrer 1998) and (Copeland et al 1998), that emerges as a proportionality law between the energy density of the scalar field and the energy density of the barotropic perfect fluid, relation that is know as an "attractor solution", with the proportionality constant m_ϕ (Liddle & Sharrer 1998), that is, $\rho_\phi = m_\phi \rho$. However the nature of the matter that corresponds to the scalar field is unknown since this matter is not detectable by the usual methods.

The research in non covariant way goes by beginning with the relation when the cosmological term Λ , often treated as a constant, having a geometrical interpretation. For explain this, the Einstein field equation is written as

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}, \quad (1)$$

where $G_{\mu\nu}$ is the usual Einstein tensor and $T_{\mu\nu}$ is the energy-momentum tensor of matter. When we take the covariant divergence of this equation, the vanishing of the Einstein tensor is guaranteed by the Bianchi identities, then it is assumed that the energy-momentum tensor satisfies the corresponding conservation law $\nabla^\nu T_{\mu\nu} = 0$, and that the covariant divergence of cosmological term must vanish, this implies that $\Lambda = \text{constant}$. Usually, this argument situates this cosmological constant on the left-hand side of the field equations, given a geometrical interpretation of the cosmological term.

However, if the cosmological term is moved to the right-hand side of the field equation, the interpretation of Λ change as part of the matter content in the following sense. The field equations now are written as

$$G_{\mu\nu} = -8\pi G \tilde{T}_{\mu\nu}, \quad \tilde{T}_{\mu\nu} = T_{\mu\nu} + \frac{\Lambda}{8\pi G} g_{\mu\nu}. \quad (2)$$

Once this is done, there is no a priori reason why this cosmological term should not vary, considering that it is the effective energy-momentum tensor $\tilde{T}_{\mu\nu}$ that satisfies the conservation law

$$\nabla^\nu \tilde{T}_{\mu\nu} = 0, \quad (3)$$

The set of equation (2, 3) and one state equation, are the fundamental tools to do the research, considering a variable cosmological term in non covariant way, because there is no a Lagrangian density that reproduce these field equations (2, 3).

In the present treatment we take into account the corresponding Lagrangian density with a scalar field

$$\mathcal{L}[g, \phi] = \sqrt{-g} \left(R - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + V(\phi) \right) + \sqrt{-g} \mathcal{L}_{\text{matter}} \quad (4)$$

where R is the Ricci scalar, $\mathcal{L}_{\text{matter}}$ correspond to a barotropic perfect fluid, $p = \omega \rho$, ρ is the energy density and p is the pressure of the fluid in co-moving frame and ω is the barotropic constant.

The corresponding variation of (4), with respect to the metric and the scalar field gives the Einstein and Klein-Gordon field equations

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\frac{1}{2} \left(\nabla_\alpha \phi \nabla_\beta \phi - \frac{1}{2} g_{\alpha\beta} g_{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \right) + \frac{1}{2} g_{\alpha\beta} V(\phi) - 8\pi G T_{\alpha\beta}, \quad (5)$$

$$\square \phi - \frac{\partial V}{\partial \phi} = 0. \quad (6)$$

From (5) it can be deduced that the energy-momentum tensor associated with the scalar field is

$$8\pi GT_{\alpha\beta}^{(\phi)} = \frac{1}{2} \left(\nabla_\alpha \phi \nabla_\beta \phi - \frac{1}{2} g_{\alpha\beta} g_{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \right) - \frac{1}{2} g_{\alpha\beta} V(\phi) \quad (7)$$

and the corresponding tensor for a barotropic perfect fluid becomes

$$T_{\alpha\beta} = (p + \rho) u_\alpha u_\beta + g_{\alpha\beta} p$$

here u_α is the four-velocity in the comoving frame. The line element to be considered in this work is the FRW

$$ds^2 = -N(t)^2 dt^2 + A(t)^2 \left[\frac{dr^2}{1 - \kappa r^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] = -d\tau^2 + A(\tau)^2 \left[\frac{dr^2}{1 - \kappa r^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (8)$$

where we identify the time transformation $N(t)dt = d\tau$, this transformation will be used in the whole work, and in special gauge we recover directly the cosmic time t .

II. FIELD EQUATIONS

Making use to the metric (8) and a comoving fluid, the equations (5) y (6) becomes (a dot mean a time derivative)

$$\frac{3\dot{A}^2}{A^2} + \frac{3\kappa N^2}{A^2} - 8\pi G\rho N^2 - \frac{1}{4}\dot{\phi}^2 - \frac{N^2 V(\phi)}{2} = 0, \quad (9)$$

$$\frac{2\ddot{A}A}{N^2} + \frac{\dot{A}^2}{N^2} - \frac{2\dot{A}\dot{N}A}{N^3} + \kappa + 8\pi G A^2 p + \frac{A^2 \dot{\phi}^2}{4N^2} - \frac{1}{2} A^2 V(\phi) = 0, \quad (10)$$

$$\left[-3\frac{\dot{A}}{A} \frac{\dot{\phi}}{N^2} - \frac{\ddot{\phi}}{N^2} + \frac{\dot{\phi}}{N} \frac{\dot{N}}{N^2} \right] - \frac{\partial V}{\partial \phi} = 0, \quad (11)$$

using the time transformation $d\tau = Ndt$, and the chain rule $\frac{\partial V}{\partial \phi} = \frac{\partial V}{\partial \tau} \frac{\partial \tau}{\partial \phi} = \frac{V'}{\phi'}$ in the last equation we obtain

$$\frac{3A'^2}{A^2} + \frac{3\kappa}{A^2} - 8\pi G\rho - \frac{1}{4}\phi'^2 - \frac{V(\phi)}{2} = 0, \quad (12)$$

$$\frac{2A''}{A} + \frac{A'^2}{A^2} + \frac{\kappa}{A^2} + 8\pi Gp + \frac{1}{4}\phi'^2 - \frac{1}{2}V(\phi) = 0, \quad (13)$$

$$\left[3\frac{A'}{A}\phi'^2 + \phi'\phi'' \right] = -V', \quad (14)$$

the Klein-Gordon equation (14) can be rewritten as

$$\frac{d}{d\tau} \left[\text{Ln} \left(A^6 \frac{\phi'^2}{2} \right) \right] = -\frac{V'}{\frac{\phi'^2}{2}}, \quad (15)$$

In the literature there are some articles where the authors try to solve these field equation in general way, for instance, in reference Chimento & Jakubi (1996), the authors present an elaborate technique for solve the Klein-Gordon equation (14), and in Reyes (2008) they use an algebraic method to obtain exact solutions taking as the basic variable the energy density of the scalar field.

In order to solve this set of equations, we introduce the ansatz, of considering that the energy density of the field ϕ is proportional to the energy density of the barotropic perfect fluid, $\rho_\phi = m_\phi \rho$, where m_ϕ is a *positive constant*. The scaling behavior occurs when $m_\phi < 1$, otherwise, the quintessence field is dominant.

The energy density and pressure of the field ϕ are given as

$$16\pi G\rho_\phi = \frac{1}{2}\phi'^2 + V(\phi), \quad 16\pi Gp_\phi = \frac{1}{2}\phi'^2 - V(\phi)$$

now, equations (12,13) are rewritten as

$$\frac{3A'^2}{A^2} + \frac{3\kappa}{A^2} - 8\pi G(\rho + \rho_\phi) = 0, \quad (16)$$

$$\frac{2A''}{A} + \frac{A'^2}{A^2} + \frac{\kappa}{A^2} + 8\pi G(p + p_\phi) = 0. \quad (17)$$

We will make now the assumption that the scalar is a barotropic fluid: $p_\phi = \omega_\phi \rho_\phi$, where ω_ϕ is a constant that play the same role of the ω parameter in the barotropic perfect fluid. Under this proposal, the field equations are

$$\frac{3A'^2}{A^2} + \frac{3\kappa}{A^2} - 8\pi G\rho_T = 0, \quad (18)$$

$$\frac{2A''}{A} + \frac{A'^2}{A^2} + \frac{\kappa}{A^2} + 8\pi Gp_T = 0, \quad (19)$$

$$\frac{d}{d\tau} \left[\text{Ln} \left(A^6 \frac{\phi'^2}{2} \right) \right] = -\frac{V'}{\frac{\phi'^2}{2}}, \quad (20)$$

where the total energy density is $\rho_T = \rho + \rho_\phi = \alpha_\phi \rho$, with $\alpha_\phi = 1 + m_\phi > 1$, and the total pressure is $p_T = p + p_\phi = \omega_T \rho_T$, with the last barotropic index is given by

$$\omega_T = \frac{\omega + m_\phi \omega_\phi}{\alpha_\phi}. \quad (21)$$

Equation (20) can be written as

$$\frac{d}{d\tau} \left[\text{Ln} \left(A^6 V^{\frac{2}{1+\omega_\phi}} \right) \right] = 0, \quad (22)$$

whose solution is

$$V(\tau) = c_\omega A^{-3(1+\omega_\phi)} \quad (23)$$

and the relation $p_\phi = \omega_\phi \rho_\phi$ implies that $\rho_\phi = \frac{2c_\omega}{1-\omega_\phi} A^{-3(1+\omega_\phi)} \sim A^{-m}$ with $m = 3(1 + \omega_\phi)$ and considering the solution to the energy-momentum tensor of the perfect fluid $\nabla_\nu T^{\mu\nu} = 0$, with solution as $\rho = M_\omega A^{-3(1+\omega)} \sim A^{-n}$, with $n = 3(1 + \omega)$, where initially the two barotropic indexes ω_ϕ and ω are different. Is know in the literature that the case $m = n$ gives an "attractor solution" and corresponds to the case when the potential of the scalar field ϕ goes to exponential behavior; this case has been studied, using other methods to understand the evolution of the universe, where this potential is introduced by hand (Lucchin & Matarrese 1985); (Halliwell 1985); (Burd & Barrow 1988); (Wetterich 1998); (Wand et al 1993); (Ferreira & Joyce 1997) and (Copeland et al 1998).

In order to solve this set of equations, we consider the case $m = n$, thus we have that $\omega = \omega_\phi$ implying that $\omega_T = \omega = \omega_\phi$ and then find the corresponding potential of the scalar field for a wide range of values of the barotropic index ω_ϕ .

This last result can be obtained making the following analysis, considering that we have three barotropic indices, two assumed, ω and ω_ϕ , and one implied, ω_T . We show now that all three must be the same. Notice that the first two equations are the standard FRW equation for the total fluid with barotropic index ω_T . Also notice that under the assumption of the relation between the density of the scalar field and the density of the perfect fluid, and the proportionality of the p_ϕ with ρ_ϕ then the potential V , $\phi'^2/2$ and all the densities (ρ, ρ_ϕ, ρ_T) are proportional to each other. From Eqs. (18,19) we now that

$$\rho_T = c_T A^{-3(\omega_T+1)}. \quad (24)$$

Then as a consequence of the proportionality between V and ρ_T the exponents in Eqs. (24, 23) should be equal, that corresponds to $m = n$ case,

$$\omega_T = \omega_\phi, \quad \Rightarrow \quad \omega = \omega_\phi = \omega_T. \quad (25)$$

III. GENERAL SOLUTION FOR FLAT SPACE

In this section we present solutions to the field equations for the flat case. Equation (20) is in the flat case written as

$$\frac{d}{d\tau} \left[\text{Ln} \left(A^6 V^{\frac{2}{1+\omega}} \right) \right] = 0, \quad \Rightarrow \quad V(\tau) = c_\omega A^{-3(1+\omega)}, \quad (26)$$

using the well known time evolution of the scalar factor for the barotropic fluid, reported in different places, in particular in reference Berbena et al (2007) that for future convenience we write as

$$A_\omega(\tau) = \begin{cases} [a_\omega \tau]^{\frac{2}{3(\omega+1)}}, & \omega \neq -1 \\ e^{2\sqrt{\frac{2}{3}}\pi G\alpha_\phi M_{-1}\tau}, & \omega = -1 \end{cases} \quad a_\omega = (\omega + 1)\sqrt{6\pi G\alpha_\phi M_\omega}, \quad (27)$$

The Hubble function $H = \frac{\dot{A}}{A} = N \frac{A'}{A}$ and the deceleration parameter

$$q = -\frac{\ddot{A} A}{\dot{A}^2} = -\frac{\dot{H} + H^2}{H^2} = -\left[\frac{A'' A}{A'^2} + \frac{A}{A'} \frac{N'}{N} \right], \quad (28)$$

can be calculated for our model, and are depending of the gauge shift function N , which should be important employing the observational data of Supernova type Ia, see references (Riess et al 1998) and (Perlmutter et al 1999).

In the gauge $N=1$, the Hubble function become

$$H_\omega(t) = \begin{cases} \frac{2}{3(\omega+1)} \frac{1}{t}, & \omega \neq -1 \\ 2\sqrt{\frac{2}{3}}\pi G\alpha_\phi M_{-1}, & \omega = -1 \end{cases} \quad (29)$$

and the corresponding deceleration parameter

$$q_\omega = \begin{cases} \frac{1+3\omega}{2}, & \omega \neq -1 \\ -1, & \omega = -1 \end{cases} \quad (30)$$

in this gauge, the deceleration parameter have a positive value for all values $\omega \neq -1$, and only in the exponential behavior have a negative value.

These results are in agreement with the solution at the deceleration parameter (28) when we take the election that this one is time dependent, (Pradhan et al 2012), or in more general sense, we can choose that $q(H) = F(H)$, where H is the Hubble function. By example, when we choose that $F(H) = -1$, we recover that the scale factor have a exponential behavior, or $F(H) = \ell = \text{constant} > 0$, we can recover the power law in the scale factor.

Employing the gauge $N \rightarrow A^3$, we obtain the following

$$H_\omega(t) = \begin{cases} \frac{2}{3(\omega+1)} [a_\omega]^{\frac{2}{1+\omega}} \tau^{\frac{1-\omega}{1+\omega}}, & \omega \neq -1 \\ 2\sqrt{\frac{2}{3}}\pi G\alpha_\phi M_{-1} e^{6\sqrt{\frac{2}{3}}\pi G\alpha_\phi M_{-1}\tau}, & \omega = -1 \end{cases} \quad (31)$$

and the corresponding deceleration parameter

$$q_\omega = \begin{cases} -\frac{5-3\omega}{2} < 0, & \omega \neq -1 \\ -4, & \omega = -1 \end{cases} \quad (32)$$

Over these last results, is worthy to mention that is necessary to choose an appropriate gauge for obtain a negative deceleration parameter in the transformed time τ .

A. Case $\omega \neq \pm 1$

The cases $\omega = \pm 1$ will be considered below, in separate way.

Following with our analysis, the temporal dependence of the potential becomes

$$V(\tau) = c_\omega \frac{1}{(a_\omega \tau)^2}, \quad \Rightarrow \quad \Delta\phi = \ell_\omega \text{Ln}(\tau), \quad (33)$$

where the constants value c_ω and ℓ_ω are determined after substitution into the complete set of Einstein equations, with the scale factor solution (27), being the general solutions for any $\omega \neq \pm 1$ the following relations are obtained

$$V_\omega(\tau) = \frac{2m_\phi(1-\omega)}{3(1+\omega)^2\alpha_\phi} \frac{1}{\tau^2}, \quad \Leftrightarrow \quad \Lambda(\tau) = \frac{m_\phi(1-\omega)}{3(1+\omega)^2\alpha_\phi} \frac{1}{\tau^2}, \quad (34)$$

$$\Delta\phi(\tau) = \sqrt{\frac{4m_\phi}{3(1+\omega)\alpha_\phi}} \text{Ln}(\tau), \quad (35)$$

when the time τ is eliminated between the two last equations we obtain the potential (or Λ) as a function of the scalar field

$$V(\phi) = \frac{2m_\phi(1-\omega)}{3(1+\omega)^2\alpha_\phi} e^{-\sqrt{\frac{3(1+\omega)(1+m_\phi)}{m_\phi}} \Delta\phi}, \quad \Leftrightarrow \quad \Lambda(\phi) = \frac{m_\phi(1-\omega)}{3(1+\omega)^2\alpha_\phi} e^{-\sqrt{\frac{3(1+\omega)(1+m_\phi)}{m_\phi}} \Delta\phi}. \quad (36)$$

The corresponding scalar potential that emerges from the temporal solution has, as is cited in the literature, an exponential behavior, (Lucchin & Matarrese 1985); (Halliwell 1985); (Burd & Barrow 1988); (Wetterich 1998); (Wand et al 1993); (Ferreira & Joyce 1997) and (Copeland et al 1998). We observe here that for all values of the barotropic parameter we have a decreasing cosmological function in time (remember the relation between the potential energy and the cosmological term, $V(\tau) = 2\Lambda(\tau)$), and that as function of the scalar field we have an exponential.

B. case $\omega = -1$.

In this case, the barotropic equation of state implies that the scalar field is constant as is the potential and we are back to $\Lambda = \text{constant}$ case, also the total energy density is constant and we have the exponential expansion factor of equation (27).

C. Case $\omega = 1$

In this case, we obtain in our proposal that $V(\phi) = 0$ and then we have a free scalar field that is the simplest case of the k-essence theory or Saéz-Ballester, see reference Sabido et al (2010) and references therein for complete solutions for FRW cosmological model and Socorro et al (2014) for the corresponding Bianchi type I anisotropic cosmological model.

In what follows we consider particular cases, divided in two branches of the barotropic parameter $0 < \omega < 1$, and $-1 < \omega < 0$.

IV. PARTICULAR SOLUTIONS

In this section we consider particular cases of the solutions with specific values of the barotropic coefficient that are of interest in cosmology and considering positive and negative branches.

A. Positive branch: $0 \leq \omega < 1$.

The relation between the kinetic term and the potential energy of the scalar field is

$$\frac{1}{2}\dot{\phi}^2 = \frac{1+\omega}{1-\omega}V(\phi) \quad (37)$$

and we have the following particular case for the barotropic parameter that are of interest in cosmology and astrophysics.

1. Dust scenario, $\omega = 0$.

The set of equations (34,35,36) have the following form

$$V_\omega(\tau) = \frac{2m_\phi}{3\alpha_\phi} \frac{1}{\tau^2}, \quad (38)$$

$$\Delta\phi(\tau) = \sqrt{\frac{4m_\phi}{3\alpha_\phi}} \text{Ln}(\tau). \quad (39)$$

$$V(\phi) = \frac{2m_\phi}{3\alpha_\phi} e^{-\sqrt{3\left(1+\frac{1}{m_\phi}\right)}\Delta\phi}. \quad (40)$$

together with the following scale factor

$$A(\tau) = [a_0 \tau]^{\frac{2}{3}}, a_0 = \sqrt{6\pi G\alpha_\phi M_0}. \quad (41)$$

Also this behavior is found using dynamical system and fitting that one critical point will be an attractor, obtaining that the corresponding factor λ in the exponential function $V(\phi) \approx e^{\lambda\phi}$, will be less than $-\sqrt{3}$, (Hernández-Aguayo & Ureña-López 2011). This value for the λ parameter was found using others techniques, in quantum solutions and in supersymmetric quantum solutions in quantum cosmology, for the same flat FRW cosmological model (Socorro & D'oleire 2010); (Socorro et al 2013). Chimento & Jakubi (1996) found the value $\lambda = -\sqrt{2}$, for inflationary era, solving the Einstein field equations as power law.

Is common say that when the scalar potential have a exponential behavior as in this case, the universe must have a fast growing scale factor. Remembering that $\alpha_\phi = 1 + m_\phi$, and considering that ordinary matter is only 4% of the total, and assuming that the scalar field account for the remaining density, we need that α_ϕ near to 18 when we consider the dark energy scenario. In that case, the scale factor is fast growing and the quintessence field is dominant in the evolution in the universe.

Comparing the different scale factor in the figure 1, the temporal potential term to the left graphics, have a big negative slope, making that the universe roll faster, having a fast growing when the parameter $m_\phi = 18$, in other values this slope is attenuated, making a moderate expansion in the scale factor.

In particular, in the temporal potential field there is a different behavior in the axe m_ϕ in the $[0, 1]$, region where the ordinary matter have a dominant behavior; in the other region, the quintessence field is dominant in the evolution of the universe.

2. Radiation, $\omega = \frac{1}{3}$

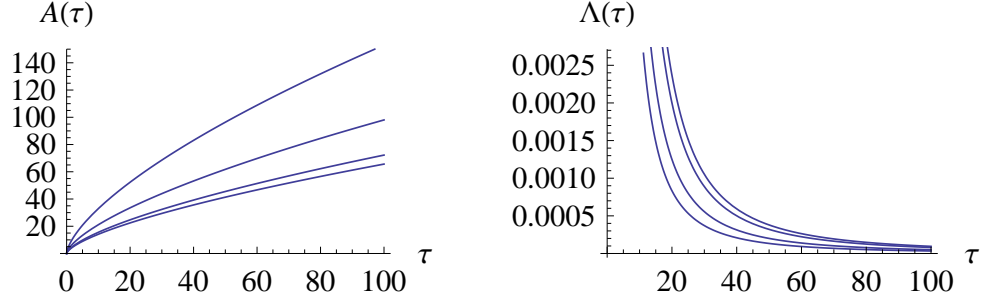


Figure 1: In the dust scenario, the scale factor has a fast growing for big values to the m_ϕ parameter, in the plot we choose the values 0.5,1,4 and 18, going to the down to up side, respectively, in the graphics. This behavior corresponds to the cosmological term (potential term), in the right to left side in the corresponding plot.

The set of equations (34,35,36), for this value of the barotropic parameter, have the following form

$$V_\omega(\tau) = \frac{m_\phi}{4\alpha_\phi} \frac{1}{\tau^2}, \quad (42)$$

$$\Delta\phi(\tau) = \sqrt{\frac{m_\phi}{\alpha_\phi}} \text{Ln}(\tau), \quad (43)$$

$$V(\phi) = \frac{m_\phi}{4\alpha_\phi} e^{-2\sqrt{1+\frac{1}{m_\phi}}\Delta\phi}. \quad (44)$$

and the corresponding scale factor

$$A_{\frac{1}{3}}(\tau) = \left[a_{\frac{1}{3}} \tau \right]^{\frac{1}{2}}, \quad a_{\frac{1}{3}} = \frac{4}{3} \sqrt{6\pi G \alpha_\phi M_{\frac{1}{3}}}.$$

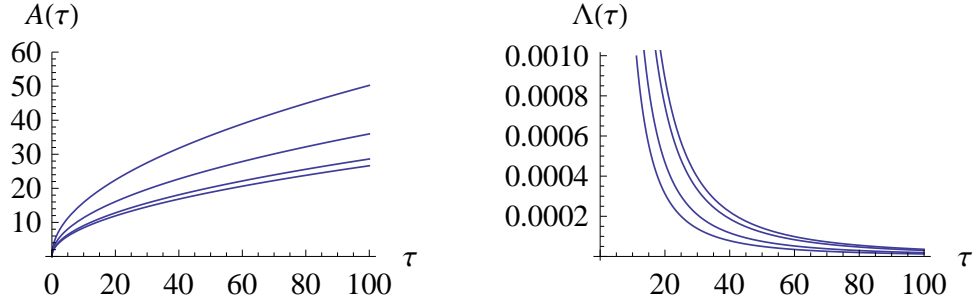


Figure 2: In the radiation scenario, the scale factor has a fast growing for big values to the m_ϕ parameter, in the plot we choose the values 0.5,1,4 and 18, going to the down to up side, respectively, in the graphics. This behavior corresponds to the cosmological term (potential term), in the right to left side in the corresponding plot.

The temporal dependence of the cosmological term goes as $\frac{1}{\tau^2}$, a result that is reported in all references that used a proportional relation between the energy density of the scalar field and the energy density to the barotropic perfect fluid, in non covariant theory.

B. Negative branch: $-1 < \omega < 0$.

In this case we write the relation between the field pressure and density as

$$p_\phi = -|\omega|\rho_\phi; \rightarrow \frac{1}{2}\phi'^2 = \beta_\omega V(\phi), \quad \beta_\omega = \frac{1 - |\omega_\phi|}{1 + |\omega_\phi|} \quad (45)$$

and we consider two particular values of ω

1. For instance, when we choose the case $\omega_\phi = -\frac{2}{3}$, i.e., $|\omega_\phi| = \frac{2}{3}$,

The set of equations (34,35,36) have the following form

$$V_\omega(\tau) = \frac{10m_\phi}{\alpha_\phi} \frac{1}{\tau^2}, \quad (46)$$

$$\Delta\phi(\tau) = 2\sqrt{\frac{m_\phi}{\alpha_\phi}} \text{Ln}(\tau), \quad (47)$$

$$V(\phi) = \frac{10m_\phi}{\alpha_\phi} e^{-\sqrt{1+\frac{1}{m_\phi}}\Delta\phi}, \quad (48)$$

with the law for the scale factor

$$A_{-\frac{2}{3}}(\tau) = \left[a_{-\frac{2}{3}}\right]^2 \tau^2, \quad a_{-\frac{2}{3}} = \frac{1}{3}\sqrt{6\pi G\alpha_\phi M_{-\frac{2}{3}}} \quad (49)$$

We consider that in this phenomenological scenario, the values of the m_ϕ are in the interval $(0, 1]$, for instance when $m_\phi = 1$, we recover the potential for the scalar field given by Chimento & Jakubi (1996) with the corresponding scale factor.

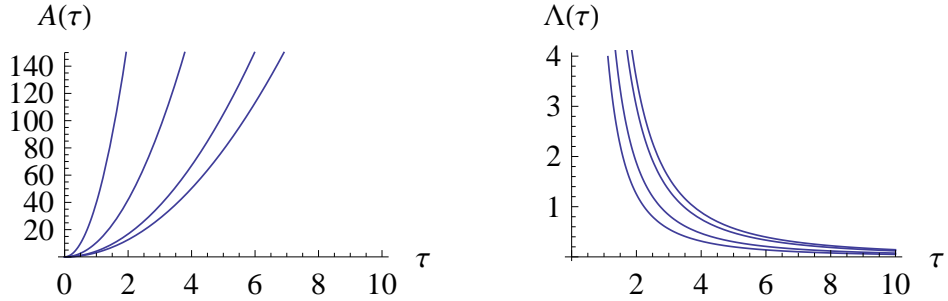


Figure 3: In the inflation like scenario, the scale factor has a fast growing for big values to the m_ϕ parameter, in the plot we choose the values 0.5,1,4 and 18, going to the down to up side, respectively, in the graphics. This behavior corresponds to the cosmological term (potential term), in the right to left side in the corresponding plot.

2. When we choose $\omega = -\frac{1}{3}$, i.e., $|\omega_\phi| = \frac{1}{3}$, we have

$$V(\tau) = \frac{2m_\phi}{\alpha_\phi} \frac{1}{\tau^2}, \quad (50)$$

so, the scalar field is

$$\Delta\phi = \sqrt{\frac{2m_\phi}{\alpha_\phi}} \text{Ln}(\tau), \quad (51)$$

thus, we can write $V(\phi)$

$$V(\phi) = \frac{2m_\phi}{\alpha_\phi} e^{-\sqrt{2\left(1+\frac{1}{m_\phi}\right)}\Delta\phi}, \quad (52)$$

with a linear evolution for the scale factor

$$A_{-\frac{1}{3}}(\tau) = a_{-\frac{1}{3}} \tau, \quad a_{-\frac{1}{3}} = \frac{2}{3} \sqrt{6\pi G \alpha_\phi M_{-\frac{1}{3}}}. \quad (53)$$

V. CONCLUSIONS

In this work we have characterized the cosmological term $\Lambda(\tau)$ as proportional the potential for the scalar field. Assuming a proportionality between the energy density of the scalar field and the the density of a barotropic fluid of the matter content, an also assuming that the pressure and density of the scalar field satisfy a barotropic law, so that the field equation in the case of the FRW metric reduces to the standard cosmology in term of a total energy density and pressure that also satisfy a barotropic law. We found that for consistency all the different barotropic parameters should be the same. In the case of flat space we were able to find general exact solutions. A common characteristic of all the solutions presented here is that the dynamic cosmological "constant" is decreasing in time as $\frac{1}{\tau^2}$ and that it is an exponential function of the scalar field. We also found the exponential behavior in the scalar field in the evolution of the universe, and in particular case, the dust era $\omega_\phi = 0$, the scalar potential have a time dependence in agreement with others results that uses directly quintessence field in the dark energy frame, signal that the universe must have a fast growing scale factor. This fast growing scale factor correspond to $m_\phi > 1$, that is when the quintessence field dominates in the universe, by instance we claim that if the percent to usual matter becomes as 4%, in the dark energy and dark matter scenario we have that α_ϕ is near to 18. However the case $m_\phi < 1$ corresponds to scaling behavior with the usual matter. In all epochs analyzed in this work using the relation between the energy density of the scalar field and the energy density of the ordinary matter, the behavior of the cosmological term goes as $\frac{1}{\tau^2}$, these results were found by other authors in a non covariant way (Chen & Wu 1990); (Abdel 1990); (Pavon 1991); (Carvalho et al 1992); (Kalligas et al 1992); (Lima & Maia 1994); (Lima & Carvalho 1994); (Lima & Trodden 1996); (Arbab & Abdel 1994); (Birkel & Sarkar 1997); (Silveira & Waga 1997); (Starobinsky 1998); (Overduin & Cooperstock 1998); (Vishwakarma 2000,2001); (Arbab 2001,2003,2004); (Cunha & Santos 2004); (Carneiro & Lima 2005); (Fomin et al 2005); (Sola & Stefancic 2005,2006); (Pradhan et al 2007); (Jamil & Debnath 2011) and (Mukhopadhyay 2011). We consider that this behavior is dependent on the relation between the energy densities considered in this work and others.

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